

Bremsstrahlung from colour charges as a source of soft particle production in hadronic collisions

A.Bialas and M.Jezabek

M.Smoluchowski Institute of Physics

Jagellonian University, Cracow*

Institute of Nuclear Physics PAN, Cracow[†]

AGH - University of Science and Technology, Cracow[‡]

February 1, 2008

Keywords: particle production, colour charges, gluon exchange, hadronic bremsstrahlung

PACS: 13.85.Hd, 13.85.Ni

Abstract

It is proposed that soft particle production in hadronic collisions is dominated by multiple gluon exchanges between partons from the colliding hadrons, followed by radiation of hadronic clusters from the coloured partons distributed uniformly in rapidity. This explains naturally two dominant features of the data: (a) The linear increase of rapidity spectra in the regions of limiting fragmentation and, (b) the proportionality between the increasing width of the limiting fragmentation region and the height of the central plateau.

*Address: Reymonta 4, 30-059 Krakow, Poland; e-mail: bialas@th.if.uj.edu.pl;

[†]Address: Radzikowskiego 152, 31-342 Krakow, Poland

[‡]Address: Faculty of Physics and Nuclear Techniques AGH, Kawory 26a, 30-055 Krakow, Poland

1. Recently, a substantial evidence is accumulating that particle production in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions at high energy satisfies the principle of limiting fragmentation [1] in a much wider range of rapidities than originally proposed. This effect, first seen in $p\bar{p}$ [2] and in p-Emulsion collisions [3], was recently studied in $d-Au$ and $Au-Au$ interactions by the PHOBOS collaboration [4, 5].

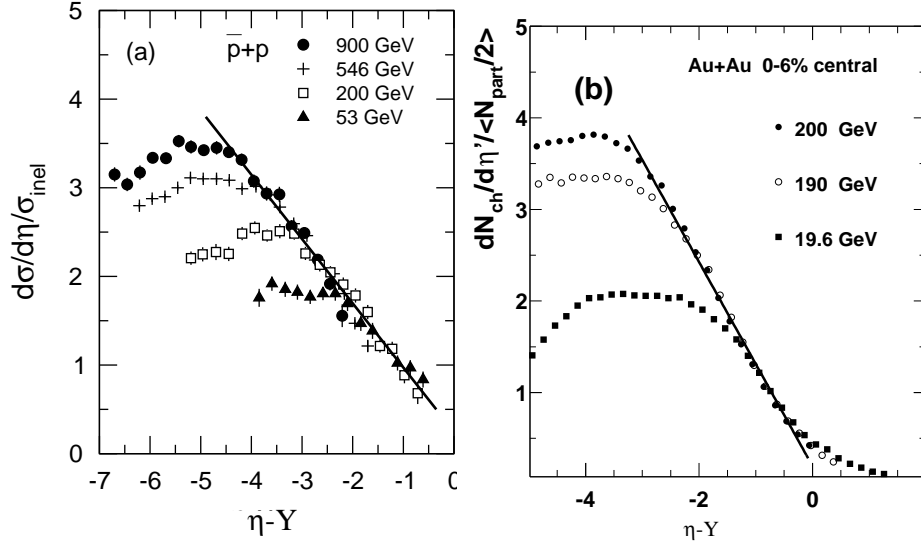


Figure 1: Particle density in pseudo-rapidity plotted versus $Y - \eta$. (a) $p - \bar{p}$ collisions [2]. (b) nucleus-nucleus collisions [5]. Lines are drawn to guide the eye.

The phenomenon is illustrated in Fig. 1, taken from [2] (1a) and [4] (1b), where particle density in pseudo-rapidity is plotted versus the difference between the beam rapidity Y and pseudorapidity η of the particle. The data of UA5 collaboration [2] on $p\bar{p}$ collisions and recent data from PHOBOS collaboration [5] on Au-Au collisions are shown. One sees three prominent features, common for the two data sets: (i) Except at very small $Y - \eta$, particle density in the fragmentation region increases *linearly* with increasing $Y - \eta$; (ii) This linear increase is followed by a "plateau" in the central rapidity region; (iii) The width of the central plateau grows with increasing Y only very slowly, if at all. This last feature implies that with increasing Y (i.e. increasing energy) the range of the limiting fragmentation region increases proportionally to Y .

It should be emphasized that these features are in blatant disagreement with the principle of boost-invariance in particle production [6]. They are thus difficult to understand in the standard description of production processes where one expects the particle density to be dominated by the central plateau [6, 7], as is common in many current models. In contrast, as is seen in the Fig. 1, the "central plateau" occupies only a fraction of the available rapidity range.

In the present note we show that all these features can be understood in a picture where particle production proceeds by a number of colour exchanges between the two sets of partons one from the projectile and one from the target. These colour exchanges lead to creation of the colour charges which emit the observed particles by the bremsstrahlung process. If the original partons in each of the colliding hadrons are uniformly distributed in rapidity (i.e. if they satisfy the Feynman dx/x rule) the resulting distribution of observed particles is linear in $Y - y$. Noting that only partons with life-time longer than the time τ_0 , needed for the colour exchange to take place, can participate in the process, we conclude that this linear increase of the spectrum must stop at a rapidity $y = y_0$, depending on the parton transverse mass μ and on τ_0 . Thus the linear increase is followed by a plateau for y smaller than y_0 . By *postulating* that this picture is valid in the c.m. frame of the collision, thus violating explicitly the boost-invariance, one accounts for the gross features of the data.

In Sections 2 and 3 a more detailed description of the model is presented. In Sections 4 and 5 the consequences of the model for particle spectrum are described. Discussion and conclusions are given in the last section.

2. We follow the standard approach to multiparticle production thus accepting that high-energy collision of two hadrons can be described by colour exchange between the partons from the projectile and from the target [8]¹.

The new idea which we propose in this note is that this mechanism is realized by colour exchange between several pairs of partons chosen at random, one from the projectile and one from the target. The members of each pair moving in opposite direction, each one radiates the observed particles (or particle clusters²) in the process of bremsstrahlung [11]. Note that by this postulate we abandon the Feynman assumption [6] that the interaction is dominated by "wee" partons, i.e. by partons with small rapidities.

¹There are many models of this type and many reviews. See, e.g. [9, 10].

²E.g. in the form of hadronic resonances.

To justify this idea we observe that it can be realized simply by one gluon exchange process. Indeed, the gluon being particle of spin one, its exchange gives the energy-independent cross-section. This means that the probability of interaction between two partons by exchanging a gluon is independent of their relative rapidity. In short, our assumption is justified by the existence of vector particles in QCD.

To obtain specific predictions one needs to know the shape of the parton distribution. We simply accept the standard idea that the partons in each of the colliding hadrons are distributed uniformly in rapidity and we can thus write their distribution in the form [11]

$$dn(z_+) = b(1 - z_+)^{b-1} \frac{dz_+}{z_+} \quad (1)$$

where b is the parton density per unit of rapidity and $z_+ = (E + p_L)/(E_i + P_i)$, with (E, p_L) are the energy and longitudinal momentum of the parton, whereas (E_i, P_i) are the energy and momentum of the beam³.

This picture makes sense only if the partons in one projectile can be treated as independent from those in the other one. This may be justified if the rapidity separation is large enough. When the rapidity separation is small, however, (i.e. in the region close to the rapidity of the center of mass) the partons from the two projectiles mix up and one cannot expect them to act independently. Also the notion of the colour separation loses the meaning. This case thus demands a special attention.

This "very central" region is best studied in the overall c.m. frame. In this frame the parton energies are not large and thus their distribution rapidly fluctuates in time. To participate in the collision process, however, the life-time of a parton must be substantially longer than the time τ_0 needed for the interaction to take place. This condition allows to estimate the effective size of the rapidity region which does not contribute to the particle production.

To see this we observe that the life-time of a high-energy parton with transverse mass equal to μ can be estimated from the uncertainty principle as

$$\tau \approx \gamma/\mu = E/\mu^2 \quad (2)$$

³These formulae apply for the right-moving system. For the left-movers one should replace z_+ by $z_- = (E - p_L)/(E_i - P_i)$.

where γ is the Lorentz factor. From the condition $\tau \gg \tau_0$ we obtain $\mu e^{y'}/2\mu^2 \gg \tau_0$ where y' is the rapidity of the parton. This implies

$$e^{y'} \geq e^{y_0} \gg 2\mu\tau_0; \quad z \geq z_0 \gg \frac{\mu^2\tau_0}{E_i + P_i} \quad (3)$$

One sees that the condition (3) restricts substantially the rapidity of partons which can participate in particle production, as one may expect τ_0 to be of the order of 1 fermi ($\tau_0 \approx 1/p_t$, where p_t is the transverse momentum exchanged in the interaction)⁴.

3. The emission of particle clusters in the bremsstrahlung process was analyzed some time ago by Stodolsky [11]. We follow his approach and write the particle distribution in the form

$$dN(x_+) = a(1 - x_+)^{a-1} \frac{dx_+}{x_+} \quad (4)$$

where a is the density of emitted hadrons per unit of rapidity and $x_{\pm} = (\epsilon \pm q_L)/(E_i + P_i)$, with (ϵ, q_L) being the energy and longitudinal momentum of the emitted cluster.

Denoting by λ the fraction of "active" partons, i.e., the partons which participated in the collision and using (1), the distribution of the bremsstrahlung products is

$$dN(x_+) = \lambda \int_{\hat{z}}^1 b(1 - z_+)^{b-1} \frac{dz_+}{z_+} \left[a \left(1 - \frac{x_+}{z_+} \right)^{a-1} \frac{dx_+}{x_+} \right] \quad (5)$$

where the lower limit of integration \hat{z} is

$$\hat{z} = \max(x_+, z_0) \quad (6)$$

with z_0 determined by the energy below which a parton does not live long enough to undergo a soft interaction and therefore also does not radiate (c.f. (3) and the related discussion in the previous section).

By changing the variables:

$$u = 1 - x_+; \quad \hat{u} = 1 - \hat{z}; \quad z_+ = 1 - ut \quad (7)$$

⁴This restriction is of course much less effective for *hard* collisions where the interaction time may be very short.

we obtain

$$x_+ \frac{dN}{dx_+} \equiv \frac{dN}{dy} = \lambda ab u^{a+b-1} \int_0^{\hat{u}/u} dt (1-ut)^{-a} (1-t)^{a-1} t^{b-1} \quad (8)$$

4. We are mostly interested in the particle distribution for rapidities outside the projectile fragmentation region, i.e, for small $x_+ \approx 0$. The formula (8) shows that we have to consider two cases.

For $x_+ < z_0$ we have $\hat{u} = 1 - z_0$. Thus, in the limit $x_+ \rightarrow 0$ we obtain

$$\frac{dN}{dy} \rightarrow \lambda ab \int_0^{u_0} \frac{dt}{1-t} t^{a-1} \quad (9)$$

One sees that the result is independent of $u = 1 - x_+$, i.e. we obtain a plateau for $y \leq y_0$.

For $x \geq z_0$ we have $\hat{z} = x_+$, i.e. $\hat{u} = u$. Consequently, (8) can be rewritten as

$$\frac{dN}{dy} = \lambda ab \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} u^{a+b-1} F(a, b; a+b; u) \quad (10)$$

where F is the hypergeometric function. Note that the result is perfectly symmetric with respect to a and b .

To see the behaviour at $x_+ \approx 0$, i.e. $u \approx 1$ we use the formula giving expansion of $F(a, b; a+b; u)$ around $u = 1$ [12]. In the limit of small x_+ this gives

$$\begin{aligned} \frac{dN}{dy} &= \lambda ab [2\psi(j+1) - \psi(a+j) - \psi(b+j) - \log x_+] = \\ &= \lambda ab [2\psi(1) - \psi(a) - \psi(b) + \log(M/m) + Y - y] \end{aligned} \quad (11)$$

where m is the transverse mass of the emitted cluster and M is the mass of the incident particle. We thus obtain a linear increase with increasing $Y - y$, as observed in the data.

The behaviour in the fragmentation region $x_+ \approx 1$, $u \approx 0$, is best seen from (10). The result is

$$\frac{dN}{dy} = \lambda \frac{\Gamma(a+1)\Gamma(b+1)}{\Gamma(a+b)} (Y - y)^{a+b-1} \quad (12)$$

which shows a deviation from the linear increase unless $a + b = 2$.

5. The distribution discussed in Section 4 can be justified for positive rapidities but is not applicable in the negative rapidity region. This is the standard problem in the bremsstrahlung model [11]. The reason is clear: Eq (4) was obtained by requiring conservation of the sum $(\epsilon + q_L)$, ignoring entirely conservation of the difference $(\epsilon - q_L)$ (i.e. ignoring entirely the target).

Since the division into positive and negative rapidities is frame-dependent, the region of applicability of (8) is also frame-dependent. To fix this, we are again forced to use the hypothesis that our considerations are valid in the c.m. frame of the collision. Of course any other frame boosted by less than y_0 is equally good.

In the actual calculations (shown in Fig. 2) we cut the distribution for negative c.m. rapidities using a simple prescription to multiply the distribution (8) by the correcting factor

$$\Phi_+(y) = \frac{x_+}{x_+ + x_-} = \frac{e^{2y}}{e^{2y} + 1} \quad (13)$$

One sees that for positive (large) y the correction factor is unimportant. On the other hand, it cuts exponentially the distribution for negative y . A similar procedure must be, of course, applied also to the other projectile, where one takes $\Phi_-(y) = \Phi_+(-y) = 1 - \Phi_+(y)$. The observed distribution is the sum of two contributions, one from the right-moving system and another one from the system moving to the left.

Since the model predicts a plateau in rapidity between $-y_0$ and $+y_0$, the exact form of the cut-off is not essential, as long as it is ineffective beyond the plateau region [11].

In Fig. 2 the calculated dN/dy is plotted versus $Y - y$ for $a = 1$, $\lambda b = 1$, $y_0 = 2$, $M = m$, two energies and three values of the parameter b . One sees that the numerical results confirm the semi-quantitative conclusions given in the previous section. One also sees that they resemble nicely the data shown in Fig. 1.

6. Several comments are in order.

(i) The idea that the colour charges created in the first step of the collision are responsible for particle production is rather general and may be implemented in many ways. An interesting possibility is to consider a more detailed model assuming that the active (radiating) partons are colour octets and the density of hadronic clusters is proportional to the local density of the

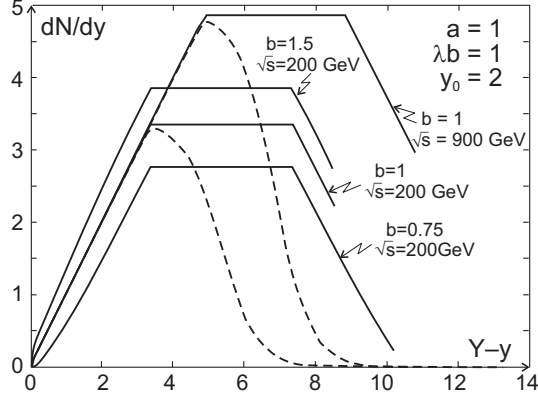


Figure 2: Particle density calculated from (8). Parameters as shown in the figure. The dashed lines show the effect of the cut-off (13)

chromoelectric field created by these partons. For a given rapidity \bar{y} there are, say, n_L octets moving to the left (i.e. having rapidities smaller than \bar{y}) and n_R colour octets moving to the right (i.e. those with rapidities greater than \bar{y}). Colour conservation implies that the representation R_L formed by left movers is conjugate to the representation R_R formed by right movers: $R_R = \bar{R}_L$

If the local energy density $\mathcal{E}(y)$ of the chromoelectric field is proportional to the quadratic Casimir operator $C_2(R_L) = C_2(R_R)$ and for $n_L \leq n_R$

$$R_L = 8 \otimes 8 \otimes \dots 8 \quad (n_L \text{ times}) \quad (14)$$

then the average energy density

$$\langle \mathcal{E} \rangle_{n_L, n_R} = \mathcal{E}_0 C_A \min(n_L, n_R) \quad (15)$$

where $C_A = 3$ is the value of the Casimir operator for the adjoint representation and \mathcal{E}_0 is a constant. The linear increase of hadron rapidity density is obtained if the rapidity distribution of radiating partons is uniform in both hemispheres. The maximum of the single particle distribution is obtained for the rapidity corresponding to $n_L = n_R$. In $p - p$ collisions this is of course the rapidity of the center of mass.

(ii) We have worked out in detail the hypothesis that the production of final hadrons from the colour charges proceeds by the bremsstrahlung process. This formulation is by no means unique. Conclusions similar to

the ones reached in this paper can be obtained if the production is described by breaking of colour strings spanned between a parton from the projectile and a parton from the target. In this case the cut-off (13) is not needed. Instead, to obtain the plateau in the central region one may postulate that the difference between the ends of a string contributing to particle production must exceed $2y_0$. This does not change the main results of the model. The essential point, needed to obtain the linear increase of the rapidity spectrum, is the flat distribution (in rapidity) of the radiating partons and the flat distribution of clusters in string decay (with energy independent density).

One should note, however, that these two versions of the model give observable differences for asymmetric heavy ion collisions (in particular p-A and d-A collisions) and for forward-backward correlations in particle multiplicities. It may thus be interesting to study these correlations experimentally.

(iii) The model contains several unknown parameters which, fortunately, have a well-defined physical meaning. Their determination from the data may thus give an interesting insight into structure of hadrons *relevant for soft interactions*. For example, a determination of the effective parton density at low momentum transfers (described by the parameters b and λ) is of clear interest. When applied to nuclear collisions, this would allow to investigate the relation between the effective parton densities and other parameters such as the number of wounded nucleons [13] and/or number of collisions.

(iv) Our argument explains only the gross features of the data. To obtain a more detailed description, it is necessary to include at least the effects of cluster decays⁵. This seems feasible, particularly in pseudorapidity, where the isotropic clusters have a well-known decay distribution.

7. In conclusion, to explain the observed strong violation of boost invariance in rapidity spectra, we have proposed a two-step mechanism for soft particle production in hadronic collisions. The first step is the multiple gluon exchange between the partons from the two colliding hadrons. In the second step, partons which were involved in this process radiate hadronic clusters. This mechanism provides a natural explanation of the observed rapidity spectra, in particular their *linear* increase with increasing rapidity distance from the maximal rapidity. Also the short plateau in the central rapidity region is naturally obtained. The scheme can be applied also to nuclear collisions and is flexible enough to account for the gross features of the data. All param-

⁵Also the change of variables ($y \leftrightarrow \eta$) may be important, particularly in the region $y \approx Y$ [14].

ters needed in this description have a well-defined physical meaning and thus their determination from the data would give useful information on hadron structure in the non-perturbative region.

Acknowledgements

Thanks are due to K.Fialkowski, W.Florkowski, K.Golec-Biernat, R.Holynski, B.Wosiek and K.Wozniak for illuminating discussions and for help in preparing the manuscript. A.B. thanks L. Mc Lerran for a comment which directed his interest to the problem treated in the present note. This investigation was supported in part by the by the Polish State Committee for Scientific research (KBN) Grant No 2 P03 B 09322 and 2 P03 B 00122.

References

- [1] J.Benecke et al., Phys. Rev., 188 (1969) 2159.
- [2] UA5 coll, G.J.Alner et al, Z.Phys. C33 (1986) 1.
- [3] R.Holynski, Habilitation thesis, Report INP 1303/PH (1986) and private communication.
- [4] PHOBOS coll., B.B.Back et al., Phys. Rev. Letters 91 (2003) 052303.
- [5] PHOBOS coll., B.B.Back et al., nucl-ex/0311009
- [6] R.C.Feynman, Phys. Rev. Lett. 23 (1969) 1415.
- [7] K.Gottfried, Phys. Rev. Letters 32 (1974) 957; and F.E.Low and K.Gottfried, Phys. Rev. D17 (1978) 2487; J.D.Bjorken, Phys. Rev. D27 (1983) 140.
- [8] F.E.Low, Phys.Rev D12 (1975) 163; S.Nussinov, Phys.Rev.Lett. 34 (1975) 1286.
- [9] A.Capella et al., Phys. Rep. 236 (1994) 225.
- [10] B.Andersson, The Lund Model (Cambridge 1998).
- [11] L.Stodolsky, Phys. Rev. Lett. 28 (1972) 60.

- [12] M.Abramowitz and I.Stegun, Handbook of Mathematical Functions (Dover, N.Y.), p.559.
- [13] A.Bialas, M.Bleszynski and W. Czyz, Nucl. Phys. B111 (1976) 461.
- [14] W.Florkowski and K.Golec-Biernat, private communication and to be published.